Determination of the radius of the Tibidabo's flying chairs using the Doppler effect.

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Abstract:

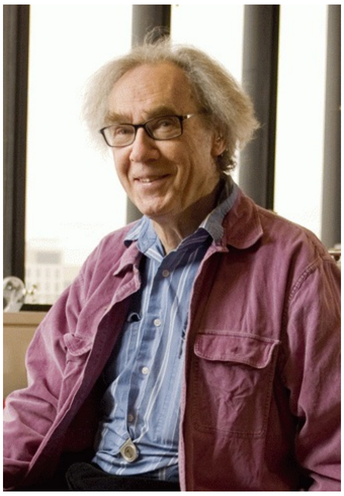
Doppler effect. That is the key word. Since its discovery in 1842, this phenomenon has revolutionized physics. It has been used to explain multiple theories, from the Big Bang theory to the universe expansion and has even been used to improve our quality of life, such as the elaboration of radars or the perfection of echocardiographies. Now, Physical Engineering 's students will use the Doppler effect in order to *take the physics out of the classrooms* and observe directly its effects on the Diavolo, one of the most emblematic attractions of the Tibidabo Park, Barcelona's biggest funfair.

Using only a microphone, a computer, a horn and a video camera, our main goal is to determinate the radius of those flying chairs.

Introduction:

The idea of this project comes from a proposal made by the Physics Engineering career's heads. We thought about many possible ideas, but the one of this concrete experiment was born thanks to a Walter Lewin's conference at the Cosmo Caixa, about "The Birth and Death of Stars". Afterwards, we were definitely convinced that the immense possibilities of the Doppler effect during "Physics 2" classes, when we studied this phenomenon.

Our experiment, detailed and explained through this document, consists on comparing the radius of the Tibidabo's flying chairs measured using the Doppler effect thanks to a registering system connected to a computer and a horn situated on one of the chairs with the real radius of the attraction, measured manually with a tape measure. In fact, we can measure frequency variations of the sonorous emission coming from the horn and use them to calculate many other parameters relative to the Diavolo. However, we will mainly focus on the radius in this paper.



1: Walter Lewin

Measuring the radius of our attraction using the Doppler effect is not so far of the techniques and methods employed by astrophysicists to calculate, for instance, the orbit's radius of a star in a binary system or to determinate black holes' mass thanks to the force they make over visible close corps. Effectively, in astrophysics, we don't have tangible tape measures to measure distances and so we must use alternative methods to determinate proportions or lengths; one of the most common ones is definitely the Doppler effect.

Through this paper, we will first present what the Doppler effect is and how it allows us, from a mathematical point of view, to calculate the radius of the flying chairs and other physical parameters. Then, we will describe our experiment, explaining the utility of each instrument we used but also showing its limitations. After that, we will expose the results we obtained and discuss the possible errors in our calculations. Finally, after concluding, we will think about other possible options we could develop directly related with the Diavolo's experiment and we will see in what measure nowadays astrophysics is similar to what we are doing…

2 The Diavolo attraction

2 Theory:

Our understanding of the Doppler Effect is our main tool when it comes to the analysis of the data obtained in the experimental procedure. That is the reason why it is necessary to explain what this effect is and how we use it in our analysis. The Doppler Effect explains how the relative speed between the source and the receptor results in a modification in the wavelength of the wave the receptor gets. In fact, either when the former approaches the latter or *vice versa* the wavelength of the wave received becomes shorter. On the contrary, when these two elements move away from each other; the wavelength measured by the receptor is larger than the emitted one. Once we know what the Doppler Effect is, we shall see how to calculate the frequency resulting from the application of this effect. This frequency is modelled by the following formula when it comes to sound waves (or other waves with a relatively low speed):

$$f^* = f\,\frac{v_s \pm v_r}{v_s \mp v_f}$$

Where:

1. f^* is the frequency received by the receptor.
2. f is the frequency emitted by the source.
3. v_s is the speed of sound (constant).
4. v_f is the speed of the source.
5. v_r is the speed of the receptor

We shall remark, though, that this formula was made according to classical physics laws, since we are measuring the frequency of the sound waves and the speeds shown are not high enough for us to take into account the relativity effects that could appear. Now we'll proceed to the demonstration of that formula:

<u>Demonstration</u>: First of all, it's well known that the wave frequency can be calculated by means of this formula:

$$f = \frac{v}{\lambda}$$

v being the wave's speed and λ its wavelength. From this point, if we take into consideration a situation where there is a source of sound waves and a receptor in relative movement, we can reinterpret this equation as:

$$f^* = \frac{v^*}{\lambda^*}$$

Where:

1. f^* is the frequency received by the receptor.
2. v^* is the wave's speed from the receptor's point of view.
3. λ^* is the wavelength received by the receptor.

Each pulse emitted by the source is given at a set time interval, but in different places. Therefore, the distance between each pulse is:

$$\lambda^* = T * (v_s \mp v_f) = \frac{1}{f} * (v_s \mp v_f);\ \text{(T is the period of the wave)}$$

depending on whether they approach or get away from each other. This distance between each pulse is equivalent to the wavelength λ^* got by the receptor.

When it comes to v^*, it represents the speed at which the receptor gets the sound wave. As the speeds treated here are really low compared to that of the light, we can neglect relativistic effects and apply the addition of velocities. Altogether the speed of the sound wave received by the receptor is:

$$v^* = v_s \pm v_r$$

In conclusion, when we substitute these last two terms, the resulting expression is:

$$f^* = f\frac{v_s \pm v_r}{v_s \mp v_f}$$

depending on whether the source and/or receptor approach each other or move away.

<u>Formulae</u>

Values we know

- ✓ f_{avg} f_{min} f_{max} *average, minimum and maximum frequencies*
- ✓ $t_{f_{min}}$ $t_{f_{max}}$ *the times at which those frequencies are obtained*
- ✓ *T: period of rotation*
- ✓ I_{avg} I_{min} I_{max} *average, minimum and maximum frequencies*
- ✓ $t_{I_{min}}$ $t_{I_{max}}$ *the times at which those frequencies are obtained*
- ✓ *"Differences" from the perfect sinusoidal form of the frequency graph, the reason being that the extremes of the function aren't at opposite angles.*
- ✓ $\gamma = \frac{I_{max}}{I_{min}}$
- ✓ v_s *: the velocity of sound through air (constant)*
- ✓ *The fact that* $h \ll R, d$ *and thus we can make the assumption that* $h \sim 0$
    - o *H: height of the plain of rotation, R: radius of the rotation, d: distance to the center of rotation*

Basic formulas known

1. $\omega = \frac{2\pi}{T}$

2. $v = \omega R = \frac{2\pi R}{T}$, *v being the linear velocity and ω the angular velocity*
3. $v = \vec{v} \cdot \vec{r}$, *the velocity of the horn that goes directly to the receiver*
4. $f' = \frac{v_s}{v_s + v} f_0$, *which comes from the Doppler formula knowing that the receptor is still.*
5. $I \propto \frac{1}{r^2}$
6. $F_{grav} = \frac{-Gm_{1m_2}}{r^2}$
7. $F_{cent} = \frac{m_2 v^2}{r}$

Basic formulas deduced

8. $f_{max} = \frac{v_s}{v_s - |\vec{v}|} f_0$, $f_{min} = \frac{v_s}{v_s + |\vec{v}|} f_0$ *obtained directly from (4)*
9. $|\vec{v}| = v_s \frac{f_{max} - f_{min}}{f_{max} + f_{min}}$, *working with equations in (8)*
10. $R = \frac{v_s T}{2\pi} \frac{f_{max} - f_{min}}{f_{max} + f_{min}}$, *making (2)=(9) and solving for R*
11. *If* $h \sim 0$ *and d>R,* $d = \frac{R(\gamma + 1 + 2\sqrt{\gamma})}{\gamma - 1}$*; using(5)*
12. $M_{black\ hole} = \frac{v^2 R}{G}$*; obtained making (6)+(7)=0*

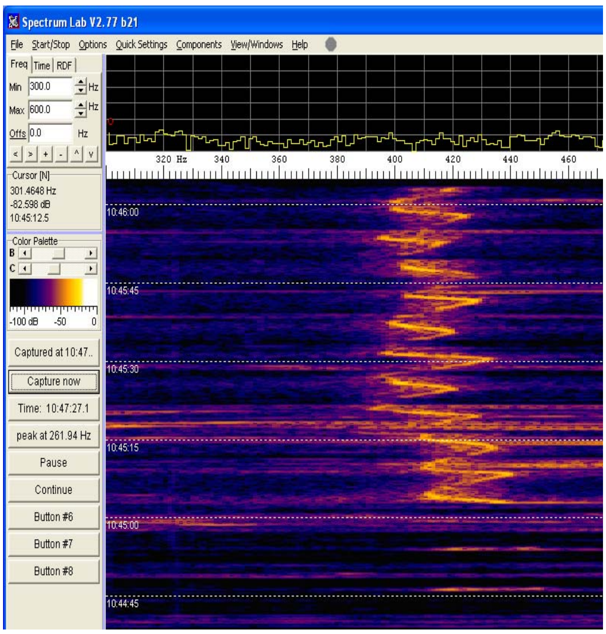
More complicated formulas deduced

13. *General* $v^2 = |\vec{v} \cdot \vec{r}|^2 = \frac{R^2 |\vec{v}|^2 cos^2\theta}{R^2 + \boldsymbol{h}^2 + d^2 + 2\sin\theta dR} [sin^2\theta + \left(\sin\theta + \frac{d}{R}\right)^2]$
14. *If* $h \sim 0$, $v^2 = |\vec{v} \cdot \vec{r}|^2 = \frac{R^2 |\vec{v}|^2 cos^2\theta}{R^2 + d^2 + 2\sin\theta dR} [sin^2\theta + \left(\sin\theta + \frac{d}{R}\right)^2]$

3. Experiment description:

A few days before taking any experiment, we visited the Tibidabo Park to familiarize ourselves with the attraction of Diavolo.

We noticed that the chairs rise several meters before starting to turn, and a couple more when turning, due to the centrifugal force produced by circular motion. Therefore, it is important to find a place to put the microphone, which collects measurements, on the same rotation plane of the attraction. Once we found the right place to put us and the equipment, we found a problem: the attraction makes small oscillations in the plane of rotation of the chairs that move them up and down, that cannot be eliminated, but we will consider them negligible.



3: Screen capture

The day of the experiment, the first thing we do is using a tape to measure the maximum radius of the Diavolo's outer seats when in operation. Meanwhile we install the computer and the microphone, with which all the data will be collected, on the proper site.

One member of the team goes to one of the outer chairs of the attraction with a horn gas, and helmets for the ears. When the chairs have gone around a few times and have reached their maximum height, this member blows the horn for about 6 laps. In this first proof he tries to aim, as far as possible, at the microphone. The most satisfying is to see at the computer how measurements vary periodically with every turn of the attraction (look at the picture 3). However, it appears a new problem. When the horn is sounding continuously for several seconds, the metallic container containing the gas cools so much until freezing moisture around it. From now on, the member on the attraction will get on insulating gloves for safety reasons. The reason for this is that the horn works with compressed gas at high pressure in a liquid state, and makes it sound due to this gas expands causing the noise. This change is an expansion of state (liquid to gas) against a constant pressure (atmospheric pressure) and it is a highly endothermic process that requires energy from the environment. This phenomenon also brings changes in our measures. After observing the data for all measures it seems that the frequency of the horn varies linearly when it is being used continuously. It is a very light gradual change and we will correct it afterwards.

For the next test we introduce a change. The fact of trying to point the microphone always involved some changes in the intensity of the received sound. Although the frequency measurements do not seem affected by this (note that the frequency is independent of intensity) in the next round the horn will be still always pointing outwards.

The first thing we notice is that the Doppler Effect can be perceived very clearly, even more than in the first test. Measurement of frequency seems to be good, the sinusoidal periodic variation also appears but at first glance do not seem as precise as in the first test.

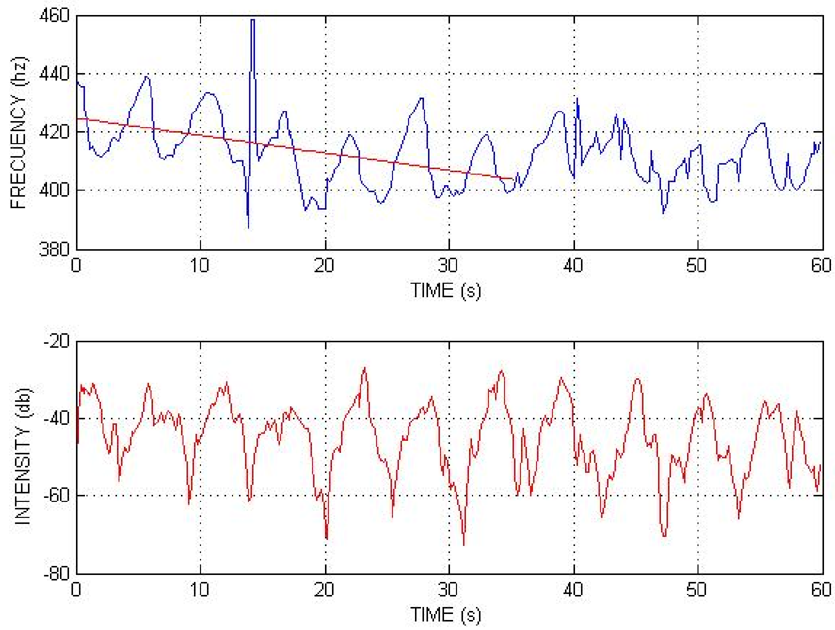
For the third test our idea was pointing the horn up. By this, the angle between the direction of emission of the horn and the imaginary line that connects it with the microphone is constant (always ninety degrees). The problem is that this type of horn only works properly in vertical position, because horizontally it cannot expel the gas properly. So we decided to make a last test like the previous one, pointing always outwards of the attraction.

Once all the data and measurements are collected, we have to interpret all the information we have to try to calculate the radius of the Diavolo attraction and everything possible.

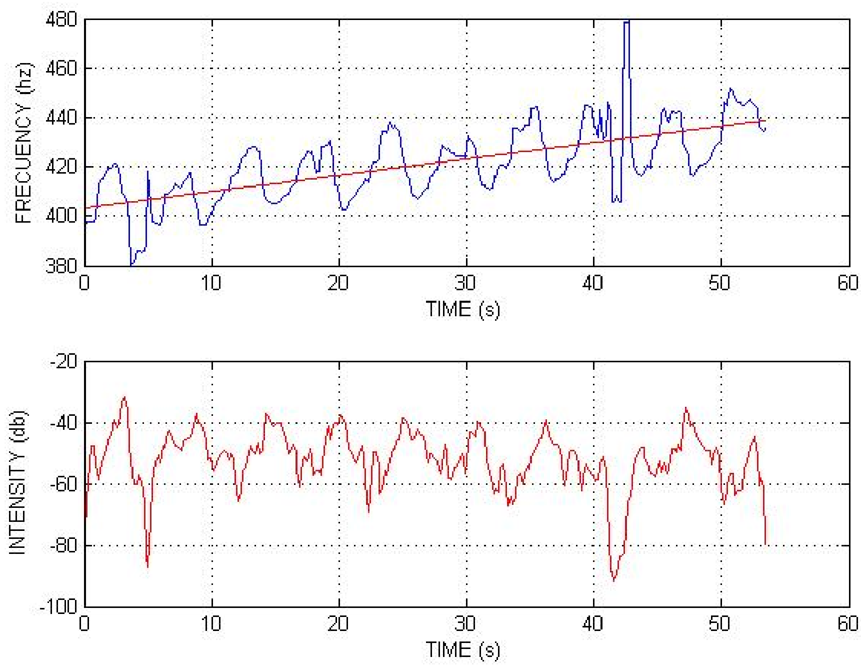
4. Results:

The following graphics show the data collected along our experiment in terms of frequency and intensity:

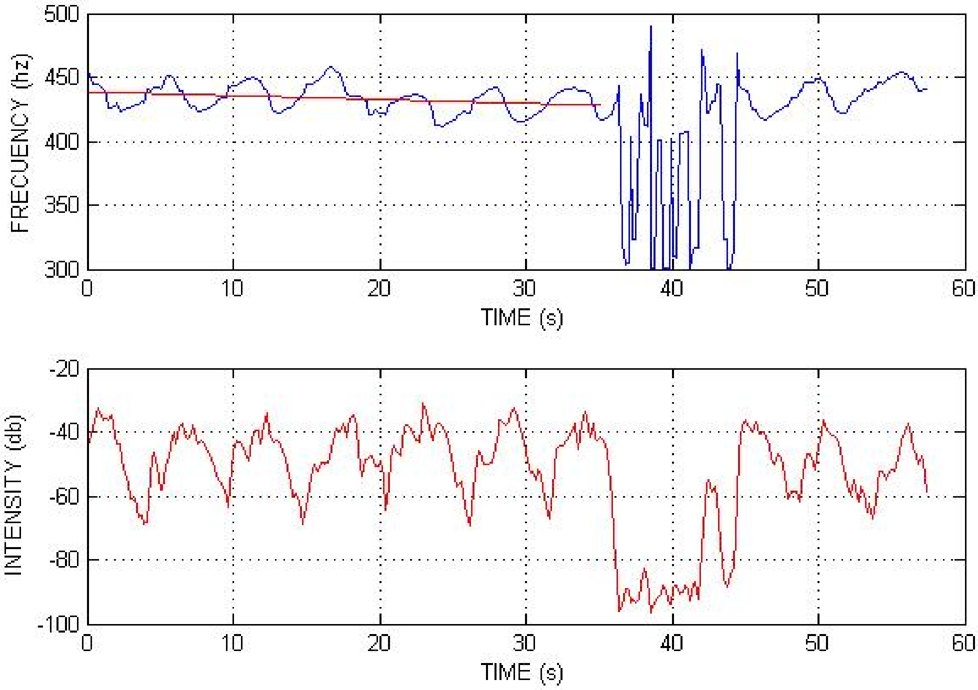
First measure



Second measure

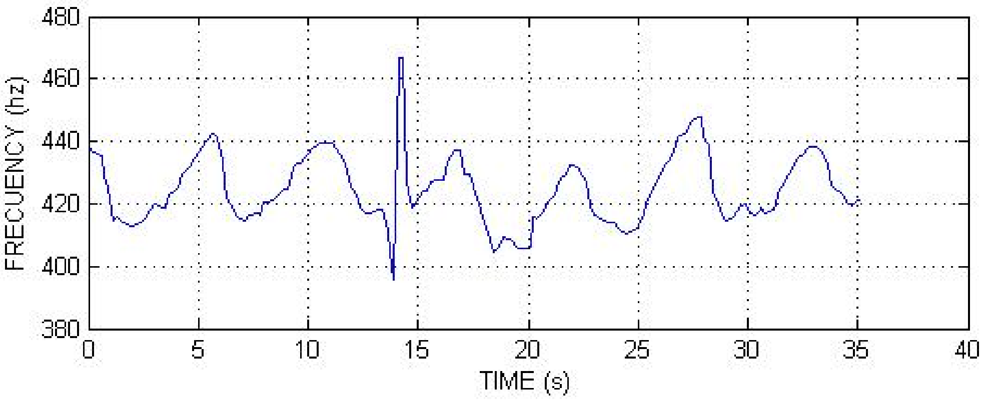


Third measure

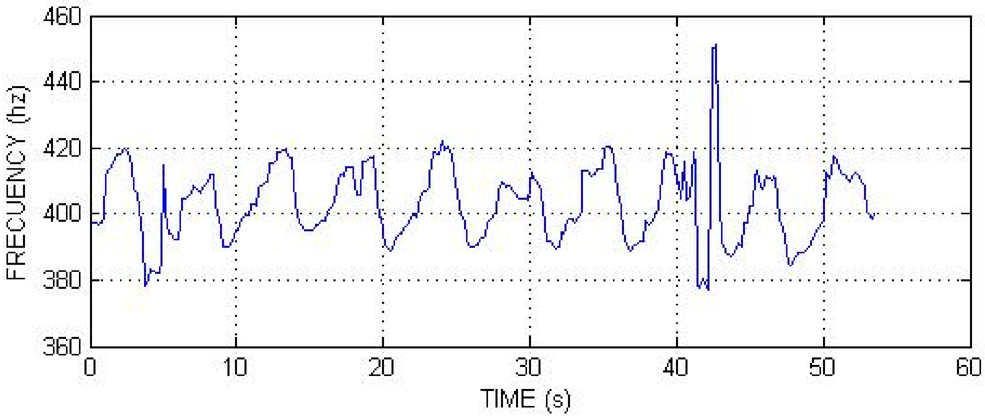


In these three images there is clearly a sinusoidal behaviour of the frequency along the time elapsed. Apart from this behaviour, we can see how the average frequency increases and diminishes regularly. This phenomenon is due to the fact that we are using our horn for a long time, which makes the characteristics of the sound emitted change. In order to correct the disruption on our measures created by the horn's behaviour, we attempt to do a lineal regression of all the measures recollected in every experiment (we assume that the variation of the average frequency along the time elapsed); this regression is represented by the red line shown in the graphics. In addition, as we can observe in the first and third measure, we only take frequency values the first 35 seconds of each experiment due to the fact that the values noted from that point are no longer reliable. The corrected graphical representations of our experimental measures in the reliable elapse of time exposed before are the following:

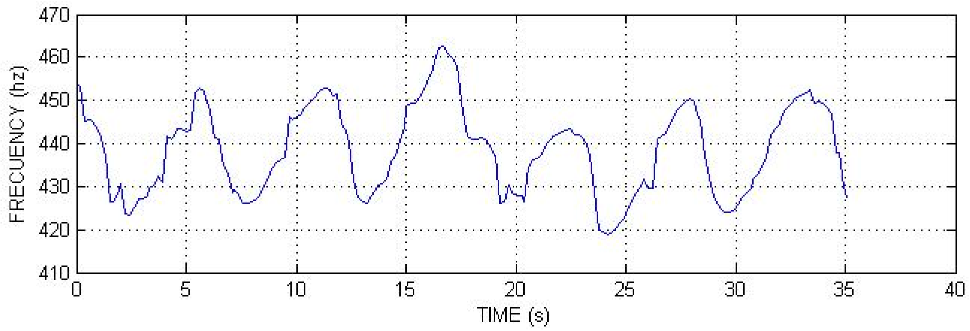
1st measurement



2^{nd} measurement



3^{rd} measurement



Once the necessary corrections were done, we decide to work with the most reliable and regular data: the measures taken in the first and third experiments.

Firstly, though we had already measured it manually with a chronometer, we want to calculate the period of the flying chairs by means of the graphics we have obtained from the experimental measures too. In order to do this, we take the time between each consecutive local maximum and local minimum and multiply it by two. Then, we make. the arithmetic mean among all the obtained values and calculate the standard deviation as the error measurement:

Period = 5,3 ± 0’35 seconds

From now on, we treat the information from each measure apart from the others. Firstly, we make list of the frequency maximum values in order to do the arithmetic mean. Next, we follow a similar procedure with the frequency minimum values. When it comes to the first measurement, we attempt to eliminate those maximum and minimum values which show an irregular behaviour, because of external issues, when compared to the rest of the data. The new value of the standard deviation which will be taken as the error measurement is:

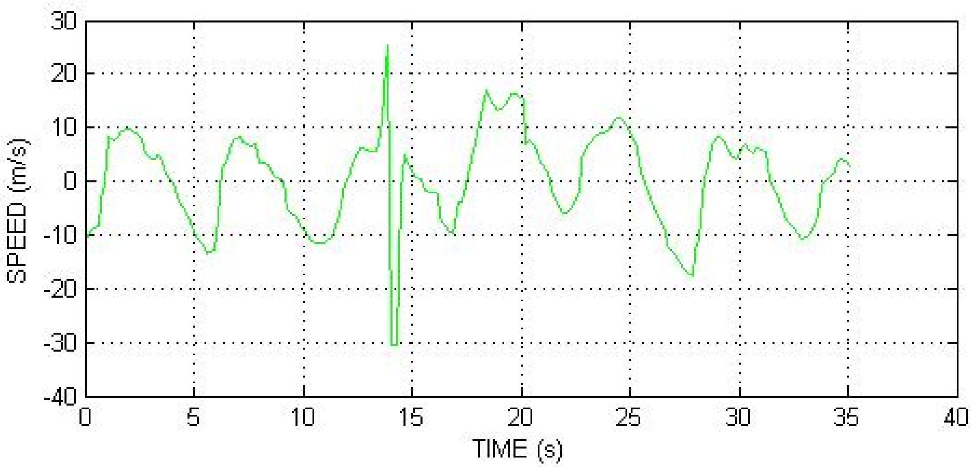
	Maximum Frequency	Minimum Frequency
Measure 1	439,4± 2,1	413,2 ± 2,05
Measure 3	452,2± 6,16	425,2 ±1,4

Then, these values will allow us to apply the formula (9) assumed from the Doppler Effect in order to calculate the rotation lineal velocity of the attraction. However, we will attempt to calculate directly the horn's velocity (source of sound waves) at each moment by means of the Doppler Effect formula (4):

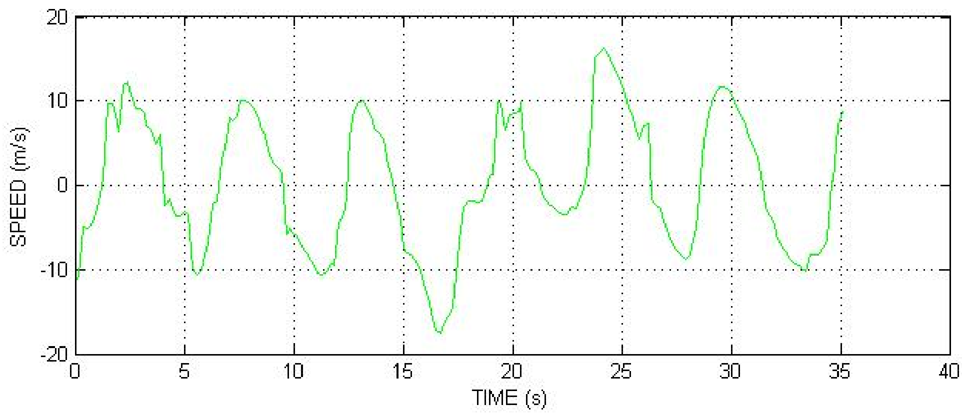
$$f^{'} = \frac{v_s}{v_s + v} f_0 \Rightarrow v = \frac{v_s (f_0 - f^{'})}{f^{'}}$$

The physical interpretation of the calculated velocity is that at which the sound source (the horn) approaches the receptor. Therefore the negative velocities represent whenever the horn gets further from the microphone. The respective graphical representations to each measurement are:

First



Third



Even though we have this huge amount of data about the source's velocity, we only care about its maximum and minimum values. The reason behind this fact is that we need the lineal velocity of the flying chairs so as to calculate the radius of the flying chairs. The maximum and minimum values shown in the graphical representation represent the moment when the chair's lineal velocity vector, tangent to their circular trajectory, points directly to the microphone.

Generally, the velocity calculated in the previous graphics (v) from the frequency represents:

$v = v_{source} \cdot \cos(\alpha)$

Where:

1. α = angle between the lineal velocity and the imaginary line that goes from the horn to the microphone.
2. v_{source} = the chairs' lineal velocity.

Taking account of the facts exposed before, it is clear that the points which we are interested in are those where a = 0º, 180º.

Now that we know the reason why we only need the maximum and minimum values from the graphics, we can use these to apply the expression (9) with the frequency values calculated previously:

$$|\vec{v}| = v_s \frac{f_{max-f_{min}}}{f_{max} + f_{min}}$$

	First measurement (m/s)	Third Measurement (m/s)
Speed	10,45	10,46

Afterwards, we calculate the error in the velocity value obtained by means of the maximum and minimum frequencies' errors according to the following expression:

$$\frac{2\sqrt{(f_{\max}\cdot\Delta f_{\min})^2 + (f_{\min}\cdot\Delta f_{man})^2}}{(f_{\min} + f_{\max})^2} v_s$$

According to which values Δf_{max} and Δf_{min} we consider (always from the standard deviation σ) we obtain the following error values:

	First Measurement (m/s)	Third Measurement (m/s)
1σ	1,17	2,38
0,5σ	0,59	1,19
2σ	2,34	4,76
0,33σ	0,39	0,79

From all the results in the last table, we will assume our error measure is that of 1σ. Altogether our results are:

	First Measurement (m/s)	Third Measurement (m/s)
Velocity	10,45±1,17	10,46±2,38

Finally, as we already have the value of the linear velocity of the flying chairs, the only thing left to do is to apply the relation (3) and isolate the radius::

$$v = w \cdot R = \frac{2\pi R}{T} \Rightarrow R = \frac{v \cdot T}{2\pi}$$

The results obtained are the following:

	First Measurement (m)	Third Measurement (m)
Radius	8,82	8,81

In this case, the radius' error is given by the expression:

$$\sqrt{\frac{4[(f_{\max} \cdot \Delta f_{\min})^2 + (f_{\min} \cdot \Delta f_{\max})^2]}{(f_{\min} + f_{\max})^2} \cdot \frac{T}{(2\pi)^2} + \frac{v^2 \cdot \Delta T^2}{(2\pi)^2}} = \sqrt{\left(\frac{\Delta v \cdot T}{2\pi}\right)^2 + \left(\frac{\Delta T \cdot v}{2\pi}\right)^2}$$

Again, according to which value of the standard deviation we take for each interfering parameter, we will obtain the following error values for the radius:

	First Measurement (m)	Third Measurement (m)
1σ	1,15	2,09
0,5σ	0,57	1,05
2σ	2,30	4,18
0,33σ	0,38	0,69

Therefore, the final values we get of the Diavolo's radius are:

	First Measurement (m)	Third Measurement (m)
Radius	8,82 ± 1,15	8,81 ± 2,09

Finally, the last thing to do is to compare these two values with the one we obtained manually with the tape measure that is:

9,5 ± 0,3 m

Seeing this comparison, we can conclude that both the experiment and the calculations derived from it were a real success!

5. Conclusions

To sum up, this experiment was really productive and enriching for all of us. It permitted us to have a different approach of science and research and also allowed us to obtain gratifying results. We have been able to determinate the Diavolo's radius by two different ways: one elemental using a tape measure and the other one calculating it by observing and measuring frequency variations of a horn emitting an approximately continuous sound.

However, many other magnitudes related with the Diavolo can be calculated as we can see below.

In spite of the material's limitations we had to overcome and the problems we had to face, this experiment really shows that the Doppler effect has many other possible applications, not only at a macroscopic scale but also at an astronomic scale. The traditional graduated tape measures loses its usefulness for big distances but the astrophysics continues progressing without limits, allowing us to measure every day more and more things, with more precision and more precision.

6. Amplification

Ideas going much further/ theoretical ideas

1. Using the fact that the graph for the frequency is not a perfect sine we can determine the distance to the attraction if we suppose $h \sim 0$. This is done by substituting for all known values in (14) and binary searching *h* comparing it with the experimental graph.
2. From the frequency we can obtain $v = \vec{v} \cdot \vec{r}$ *at* each instant. This, as *(13)* shows, only leaves 2 candidates for θ which can be reduced to 1 knowing the intensity (or alternatively knowing $\frac{dv}{dt}$). Once *d* and *R* are known, we can parameterize the circular trajectory and thus obtaining the position of the horn in a 2-D space at each instant.
3. Combining the equations (11) and (14), i.e. both the intensity and the frequency, we could try to obtain all h, d, R. Alternatively we could use equation (11) and the graph of $\frac{dI}{dt}$.
4. Using the previous idea the same of idea 1 could be done just by adding the height and thus obtaining a 3-D position at each instant.

5. Another way of obtaining a 3-D position without making the assumption that we have a circular trajectory (and that it could be an elliptical one) could be to take different measurements from different places. This could be a small analogy of trying to obtain the position of a satellite.
6. Reverse engineer the process and trying to obtain the position of an emitter on the surface by taking simultaneous measures from different chairs. This could be an analogy of the GPS system.

Viable continuations for the experiment

1. Using the drop in the intensity graph, knowing *d* and using basic trigonometry we can approximate the radius of the pivot in the centre of the attraction
2. Supposing $h{\sim}0$ calculating *d* from *R* using (11)
3. Calculating *h* from the change in intensity while the chairs are going up.
4. Calculating the mass of the hypothetical dark hole using (12)
5. Trying to parameterize the secondary rotation of the attraction and trying to account for it.
6. Showing that the relativistic effect is negligible.

7. Relation with Astrophysics

Although this project may not seem much interesting at first sight, it really is. Actually, the procedures shown in this paper hold similarities with those applied in astrophysics. However, this time we are dealing with light waves instead of sound waves, which means we are required to take into account the relativity effects that appear at high speeds near the speed of light. Even though we are not going to show or explain how to get the Doppler effect formulas used in these circumstances (as it is not the goal of this section), the appearance of relativity effects is an important detail to keep in mind. The main applications in astrophysics related with the procedures and analysis shown in this paper are:

1. The tracking of Binary Systems and the calculation of its period and frequency of rotation.

By analyzing the wavelength of a star, we can determine if it is moving away or approaching us. Consequently, whenever a certain pattern in the changes of wavelength of the light rays within a certain period is registered, there is a possibility that the star we are studying might be rotating around another star. In this case, we would have found a binary system and the Doppler effect would make possible not only finding it but determining the mass of both stars among other characteristics of the system such as the period and frequency of rotation.

2. The tracking and calculation Black Holes' mass.

The procedure followed is exactly the same as the one explained before when we explained the application of the Doppler Effect in binary systems. However, the analysis of data this time is much easier this time as the black hole has all its mass centered in a single point of space. This means that the star trajectory will be more circular and that we will be able to use the formula of the normal acceleration to calculate the Black Hole's mass.

3. Determining whether galaxies are moving away or approaching us and, in consequence, determining whether the universe is expanding or shrinking.