Determination of the Distance Travelled by a ball Dropped from the Carousel

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ABSTRACT

The purpose of this experiment was to apply Newton's laws in the carousel of Tibidabo. We calculated the distance a ball thrown from a given height should travel before reaching the floor, taking into consideration the velocity of the carousel. Despite several problems that raised during the realization of the experiment, the results were satisfactory and the ball fell nearly where expected.

MATHEMATICAL EXPLANATION

Given the kinematic equations:

$$\left.\begin{aligned} x &= x_0 + v_x \cdot t \\ y &= y_0 + v_{y0} \cdot t + \frac{1}{2} \cdot g \cdot t^2 \end{aligned}\right\}$$

Where x_0, y_0 and v_{y0} are assumed to be 0 because the object will experience free fall.

Substituting in the equations above , and knowing that the centripetal acceleration at any point of the carousel is given by $a_c = \frac{v_x^2}{R}$, where R is the radius from the center of the carousel to the point, i.e. $R = \frac{l}{2\pi}$, where l is the circumference longitude , then $y = \frac{1}{2} \cdot g \cdot \frac{x^2}{a_c \cdot R}$.

To determine the centripetal acceleration using a pendulum, the tension in the x-direction is the centripetal force, wheras the tension in the y-direction is the weight of the object. Thus, the angle it makes with the vertical is $tan\,\theta = \frac{T_y}{T_x} = \frac{m \cdot g}{m \cdot a_c} = \frac{g}{a_c}$.

Substituting all the variables calculated, the distance x the object will travel when dropped from at any point on the carousel,

$$x = \sqrt{tan\,\theta \cdot y \cdot \frac{l}{\pi}}$$

EXPERIMENTAL PROCEDURE

To do the experiment, the following material was used:

- Computer with accelerometer and software LoggerPro

- Tennis ball
- Rope
- Tape measure
- Calculator
- 2 cones and 2 sticks

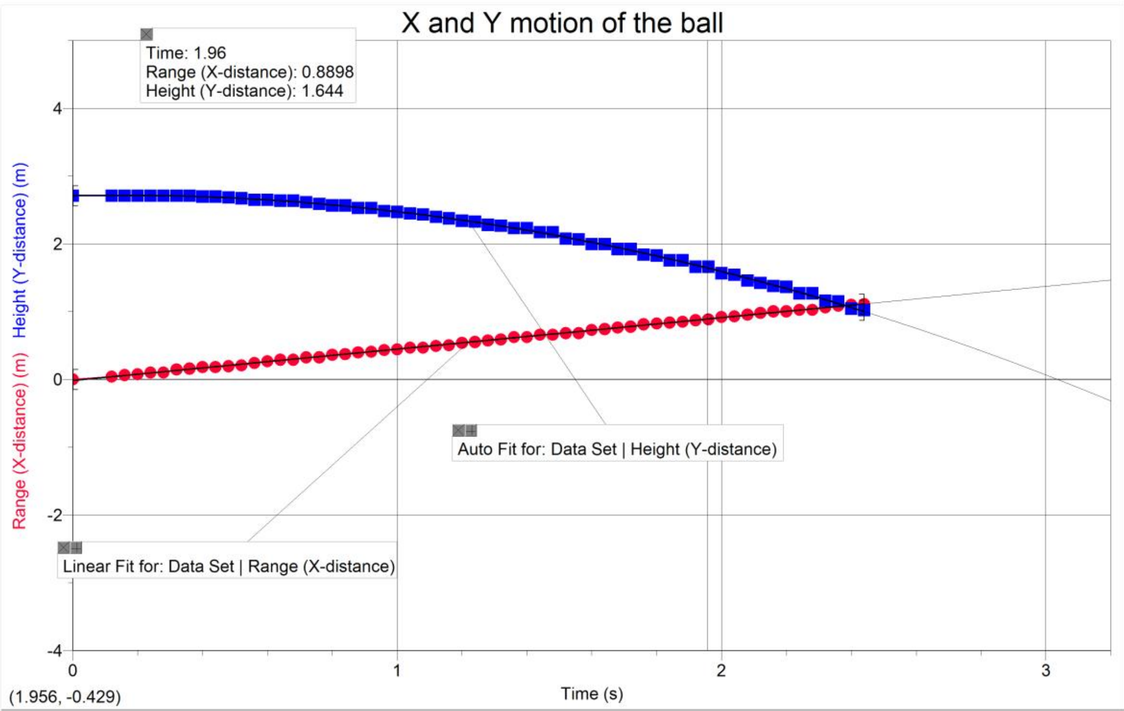
The initial idea was to measure the centripetal acceleration with an accelerometer, but a problem turned out and an inclinometer had to be used instead.

By changing the values of and y , x could be found. The outer part of the carousel was measured circling it with a rope and measuring the distance on the rope, and y was set by defining a height from which the ball should dropped. Since the carrousel had some tangential acceleration –it arrived at a maximum tangential velocity-, the ball was thrown at the moment where the centripetal acceleration was at its maximum, i.e. θ had its highest value. Taking all these into consideration, the distance was of

$$x = \sqrt{tan\, 4.5º \cdot 2.8m \cdot \frac{31.4m}{\pi}} = 1.48m$$

The cones and the sticks were put at a point outside the carrousel at a distance of 1.48m apart, and the ball was dropped at the first stick. Theoretically, in an ideal situation –with no friction, no turbulences in the carousel- the ball should touch the floor at the second stick.

RESULTS



The graphic above represents both the x and y coordinates of the parabolic motion of the ball versus time. All the successive points form a line that can be approximated using a linear and quadratic regression. As the attraction had a fence which makes it extremely difficult to see where the ball exactly fell, these regressions are only an approximation of the point where the ball touched the ground. From the plot can be deduced that when y=0.001129m (assuming y=0m), the x-component of the position of the ball equals 1.39m.

This result slightly differs from the one we calculate *in situ*, which was 1.48m.

To see how the experimental result differs from the one obtained analysing directly the video made, the relative error shows it:

$$\text{Relative error} = \frac{|\text{Experimental result} - \text{Theoretical result}|}{\text{Theoretical result}}$$

$$\text{Percentual error} = \frac{|1.48\ - 1.39|}{1.48} \cdot 100 = 6\%$$

CONCLUSION AND EVALUATION

Despite all in-field problems -inaccuracy of measurements and other factors that could have ruined the experiment-, we finally got through and the ball obeyed the calculated trajectory. In the majority of the trials, however, the ball did not reach the distance estimated, because it was dropped instants before it reached the first stick. On the contrary, when it was dropped above the first stick, it almost reached the exact distance estimated. As a further study , the fricitional force caused by air could be calculated, since the 6% of distance that the trajectory was reduced was due to such opposite force to the movement.