

# STUDY OF THE CONSERVATION OF LINEAR MOMENTUM AND THE LOSS OF KINETIC ENERGY IN A COLLISION BETWEEN TWO BUMPER CARS



**Figure 1:** Collision between two Tibidabo's bumper cars used during the experiment.

**Source:** Fisidabo videos

<https://www.youtube.com/watch?v=kG-3471ojM0>

## ABSTRACT

The objective of our project is to check if the linear momentum is conserved in a real collision and to determine if it is an elastic or an inelastic crash. We used a high-speed camera to collect data for each collision and then, that was analysed afterwards using Logger Pro. We determined that the linear momentum is almost totally conserved and that the collisions are inelastic.

## INTRODUCTION

When the Fisidabo's project was presented to us, we were studying collisions in physics class. We had learned that there are two types of collisions: the elastic and the inelastic. We were quite surprised by the fact that the first one, which is only completely true in theory problems, satisfies two equalities with the same variables related differently: the conservations of linear momentum and the conservation of kinetic energy. In the inelastic crashes, only the linear momentum is conserved.

Consequently, we decided to study to what extent the equations exposed was true in a real collision. Hence, our objective for this project was to check if the linear momentum is conserved and to determine the percentage loss of energy to be able to say if it is elastic or inelastic. If it were an elastic crash, the percentage loss of energy would be 0%.

## THEORETICAL BACKGROUND

### LINEAR MOMENTUM

The linear momentum for massive particles is a vector quantity defined as:

$$\vec{p} = m \cdot \vec{v}$$

Where  $m$  holds for the mass of the object and  $v$  for its velocity. In a closed system, a place without external forces, the sum of all the momentums is always conserved, which means that the sum of all the initial momentums equals the sum of the final momentums.

This means that:

$$\vec{p}_i = \vec{p}_f$$

Where  $i$  stands for initial and  $f$  for final.

So, for example, in a collision between two objects with masses  $M_1$  and  $M_2$  and initial velocities  $v_{1i}$  and  $v_{2i}$  and final velocities  $v_{1f}$  and  $v_{2f}$ , the expression would be:

$$\vec{p}_i = M_1 \cdot \vec{v}_{1i} + M_2 \cdot \vec{v}_{2i} = M_1 \cdot \vec{v}_{1f} + M_2 \cdot \vec{v}_{2f} = \vec{p}_f$$

In our experiment, due to the fact that we will consider both masses to be equal and those masses have not been given to us, we will calculate the linear momentum divided by the mass instead of the linear momentum. We will use the following equation:

$$\frac{\vec{p}}{m} = \frac{m(\vec{v}_1 + \vec{v}_2)}{m} = \vec{v}_1 + \vec{v}_2$$

### KINETIC ENERGY

The kinetic energy is the energy contained in the movement of a massif body. It is defined as:

$$E_c = \frac{1}{2} m \vec{v}^2$$

Where  $m$  holds for the mass of the object and  $v$  for its velocity.

Using this kinetic energy, the percentage loss of energy can be calculated by using the following equation:

$$\% \text{ loss of } E_c = \frac{E_{ci} - E_{cf}}{E_{ci}}$$

Where  $E_{ci}$  is the initial kinetic energy and  $E_{cf}$  holds for the final kinetic energy.

### COLLISIONS

There exist two types of collisions: the elastic and the inelastic. When there is no deformation in a collision we call it elastic. Consequently, linear momentum and kinetic energy are conserved. However, in a real experiment it does exist deformation and, in this way, it absorbs energy. Part of the kinetic energy is transformed into heat, avoiding the kinetic energy to be conserved, although the conservation of linear momentum is still applied. Those are the inelastic crashes.

The most inelastic crash is the one in which the two objects that collide end up with the same velocity, so the time they are together is infinite. Consequently, to have an elastic crash we should not have the two objects together -with the same velocity- for more than an instant.

Otherwise, there would be a loss of energy. Hence, the larger the time the two objects are together, the larger the loss of energy will be.

## METHOD

To carry on our experiment we have to do the following:

1. Set a unidimensional space reference system-a tape in the floor with 50cm divisions.
2. Set a camera in a position in which you can see the collision in elevation centred on the crash and being able to see the reference system.

From this point two different experiments where designed in order to study two similar situations to be able to have more solid conclusions. Those are:

### EXPERIMENT A

- a. Align two cars, facing each other on the reference line. One of them will remain at rest at the point of the collision.
- b. Accelerate the other car until maximum velocity and, just before the collision, stop accelerating.

### EXPERIMENT B

- a. Align again the two cars facing each other on the reference line. This time the collision will be in the middle of them.
- b. Accelerate the two cars until maximum velocity and, just before the collision, stop accelerating.

## RESULTS

After doing the experiment, we had to analyse the videos of the collisions using a program called Logger Pro. From it, we obtained the initial and the final velocities of the two cars in the two experiments. Those data can be seen in figure 2.

	$\vec{v}_{1i}$	$\vec{v}_{2i}$	$\vec{v}_{1f}$	$\vec{v}_{2f}$
EXPERIMENT A	0,099	0,01	0	0,081
EXPERIMENT B	0,096	-0,063	-0,105	0,044

Figure 2: Raw data

From that data we can calculate, as specified before, the momentum divided by mass, and the percentual loss of energy. As we are working in a unique dimension, momentum will not be a vector anymore, and will have arbitrary units. The results can be seen in figure 3.

	Initial momentum	Final momentum	% loss ef energy
EXPERIMENT A	0,099	0,096	24
EXPERIMENT B	-0,009	-0,019	71

Figure 3: Processed data.

## CONCLUSIONS

### EXPERIMENT A

Analysing the results obtained for experiment A, we can see a loss of energy that is significantly lower than the one in the experiment B because of the shorter duration of the collision, which is related to the absorption of energy as exposed in the theoretical background. We have also seen a small loss of the modulus of the momentum divided by mass.

### EXPERIMENT B

For our second experiment (experiment B), we can see in one hand that there is a huge loss of energy related to the large duration of the collision. It may also be due to some of the energy that could have been transformed into movement in another direction, which is not being contemplated.

In the other hand, we can see a modulus of the momentum divided by mass gain, which is physically impossible. A possible justification may be that the masses were not exactly the same -the driver of one car was significantly heavier than the other one. Hence, rearranging the results assuming that one mass was between a 7% and a 9% heavier than the other, we get a small loss of the modulus of the momentum (See figures 4 and 5 in annex 1). This loss is due to the frictional force with the floor.

### GENERAL CONCLUSION

With everything seen before, we can conclude that the longer the collision duration is, the larger the percentual energy loss will be. We can also say that elastic crashes do not exist since there are no collisions without loss of energy, as there must be at least a short moment in which the two objects are together. Related to our results, we can see that the experiment A was a 76% elastic crash and the experiment B a 29% elastic crash.

## ANNEX I

To rearrange the momentum supposing one mass between a 7% and a 9% heavier we have used the following equation:

$$\frac{\vec{p}}{m} = \frac{m\vec{v}_1 + (100 + \frac{(7 \text{ or } 9)}{100})m\vec{v}_2}{m} = \vec{v}_1 + (100 + \frac{(7 \text{ or } 9)}{100})\vec{v}_2$$

Where  $m$  is the mass of the lightest car.

The values obtained from this rearrangement are:

9% heavier	Initial momentum	Final momentum
Experiment A	-0,018	-0,015
Experiment B	0,099	0,098

Figure 4: Results obtained from the rearrangement done assuming the second car is 9% heavier.

7% heavier	Initial momentum	Final momentum
Experiment A	-0,016	-0,016
Experiment B	0,099	0,097

Figure 5: Results obtained from the rearrangement done assuming the second car is 7% heavier.

In the experiment B we can see a smaller loss of the modulus of the momentum divided by mass than the one, which was not rearranged.